# GIS, Spatial Analysis and Spatial Statistics

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## I Introduction

> Spatial analysis will be applied to an ever increasing number of application areas. GIS data manipulation tools will become ever more sophisticated and easier to use. They are already today being included in office software packages such as spreadsheets. We are rapidly approaching the time when every desktop PC will be able to perform spatial analysis.

The above quotation comes from a draft of *GI2000: towards a European geographic information infrastructure,* published during the past year by DG XIII of the European Commission (EGII, 1995), which attempts to lay the foundations of a European infrastructure for spatial information. Its content may well be a surprise to readers whose knowledge of spatial analysis ended with a practical class on the nearest neighbour statistic taken, probably unwillingly, as part of their degree studies and who thought that it had disappeared into history. Largely as a result of the growth of GIS, spatial analysis is back on the research agenda and in this year's review I will attempt to give a flavour of current work in the field.

At the outset, it is best to be clear what we mean by the term. In the GIS literature, and especially in system manuals and brochures, the view seems to be that spatial analysis is simply the general ability to manipulate spatial data using a familiar set of largely deterministic functions which includes basic spatial queries, buffering, overlay using simple map algebra and the calculation of derivatives on surfaces such as slope and aspect. This type of work can be called *spatial data manipulation* and, since it is precisely this ability to handle spatial data *spatially* that differentiates a GIS from any other database management system, it is essential to any information system claiming to be geographical. It is also what differentiates a true GIS from computer-aided design or mapping packages. In *spatial statistical analysis* knowledge of a process is used to predict the spatial patterns that might result, and the likelihood of any observed pattern

being a result of this process is then established by an analysis of one or more of its realizations. In contrast, exploratory *spatial data analysis* examines an observed distribution and attempts to infer the process that produced it. The objective is usually to find patterns in data that are meaningful in relation to the investigators' existing domain knowledge. Both are different from *spatial modelling* in which the objective is to produce realistic mathematical models of the type used in, for example, retailing (Birkin et al., 1996) and the environmental sciences (Goodchild, Parks et *al.,* 1993; Goodchild, Steyaert et al., 1996) that deterministically predict spatial pattern. In this year's review, I will be concerned mostly with exploratory spatial data analysis (ESDA) where there has been a major renaissance brought about by widespread access to very powerful computer workstations and, equally critically, to often very large, structured data of the type which are common in any GIS. This renaissance of spatial analysis has been driven by several academic changes.

First, as we saw in last year's review (Unwin, 1995), users of geographical information systems have begun to ask questions about the reliability of the results obtained from simple spatial manipulations of geographic data, such as interpolation and map overlay, and have begun to realize the importance of a statistical approach. As a result, there has been a series of discussions of the role of spatial analysis in GIS and the relationships between the two (see Goodchild, 1987; Anselin, 1989; Fotheringham, 1992; Goodchild et al., 1992; Anselin et al., 1993; Fotheringham and Rogerson, 1993; 1994; Bailey and Gatrell, 1995; Fischer et al., 1996). The greater part of this debate has addressed technical questions about how to couple GIS with the required statistical 'functionality'. With the notable exception of Anselin et al. (1993), rather less attention has been directed to what should be coupled and why.

Secondly, work by statisticians has developed a substantial body of statistical theory about spatial data to which GIS users can turn but which did not exist in geography's so-called 'quantitative revolution' (see, for example, Diggle, 1983; Ripley, 1981; 1988; Upton and Fingleton, 1985; 1989; Haining, 1990; Cressie, 1991; Walden and Guttorp, 1992). The statistical view is characterized by the notion that spatially distributed information can be regarded as the outcome of some stochastic process operating in the plane. If we can postulate the nature of the process in mathematical terms, we can deduce its spatial outcomes and examine whether or not an observed pattern is a plausible realization of it. As Harvey forcefully pointed out many years ago (Harvey, 1966), a much more difficult alternative is to identify the process and model it appropriately from the evidence of a single mapped realization, yet this is frequently what is required. Very few of these new methods of analysis are as yet implemented in existing GIS but specialized software is now readily available (Anselin, 1990; Rowlingson and Diggle, 1991; Diggle and Rowlingson, 1993). Recently INFOMAP (Bailey, 1990) has become available at very low cost and includes analysis methods such as density estimation, kriging and K-function computation, all of which were developed during the 1980s specifically to handle spatial data (Bailey and Gatrell, 1995).

Thirdly, now that spatial data are easily obtained at extremely high spatial resolution, and computing and mapping the results are unproblematic, there has been developed a series of strategies for exploratory spatial data analysis often using visualization (Unwin, D.J., 1994). This is a data and cartography driven approach to spatial analysis, concerned with the recognition and description of spatial patterns and their representation on maps. The methods employed vary from allegedly simple statistics dating from the 1950s (notably the spatial autocorrelation tests of Cliff and Ord, 1973), through

the direct use of visualization (Hasslett, Wills *et al.,* 1990; Haslett, Bradley *et al.,* 1991; Hearnshaw and Unwin, 1994) to automatic machines (Openshaw, 1994) and artificial life forms for pattern detection (Openshaw, 1995).

In the remainder of this review I will examine some of the recent approaches to exploratory spatial data analysis. First, I will look at the meaning of the central idea of *pattern* in a data-rich GIS environment. This leads naturally to a discussion of a class of *local statistics* that are rapidly gaining acceptance, and in turn leads to a consideration that is central to geography, the definition of what is, and what is not, in the locality of some place. Much of the review is concerned with the influence of developments in statistical analysis on GIS, but the review concludes by an examination of the influence of GIS on the development of spatial statistics, in particular the notion of *GISable methods* of analysis.

## II Spatial pattern, projection and process

Pattern is that characteristic of the spatial arrangement of objects given by their spacing in relation to each other. It should not be confused with the idea of dispersion, which is relative to some defining area, or with density, which is the average number of objects in a given area. Patterns might consist of clusters of points, a more-regular-than-random arrangement, trends across real and statistical surfaces and so on. Given Tobler's 'first law of geography', that near places are more likely to be related than distant ones, it is hardly surprising that most geographical patterns of interest involve groupings of similar values in clusters.

In almost all the work by statisticians, and in most of 'quantitative geography', the approach taken has been to map objects of interest using their location on the planet's surface as measured by plane Cartesian co-ordinates based on a map projection. This is, of course, a reasonable assumption for small areas of interest and it allows the use of simple geometry to find distances and areas. If large areas are studied, then referencing can be a latitude/longitude pair and in a GIS environment calculations of the real distances and areas are only slightly more difficult. There is a danger that standard functions in proprietary GIS used, for example, in interpolation or the estimation of distance functions do not recognize the need to compute the great circle distances. What is often not realized in the modelling and search for spatial pattern is that the concept must also contain some *projectional* component. Patterns come and go according to how we project the data. A simple example is the view we get of the hemispherical night sky on which we see a projection of objects distributed in at least three dimensions. Provide an extra dimension of time/space and the well-known nonrandom patterns studied by astrologers, such as The Plough, disappear. Opposite operations are also well known, perhaps the simplest being the often-cited correlation between the number of storks and births in India. Project these same data with a third axis which locates them in time and the reason for the pattern becomes obvious.

The search for the 'correct' number of dimensions in data is (was?) the subject of methods of 'factor' analysis and its more modern, computer-based alternative, called 'projection pursuit'. Within appropriate software it is a relatively easy matter for non-standard projections to be calculated and these can provide much more useful frameworks in which to display data and look for pattern. For example, Dorling (1992; 1994) has used the projection given by an area cartogram based on small area population

totals to show detailed variations in the social geography of Britain. Similarly, patterns revealed in plots of data on to empirical projections defined using multidimensional scaling have been exploited in a number of studies by Gatrell (1979; 1983; 1991).

The second component of a pattern is that given by the process which generated it. Such a process could be totally deterministic, with a single unique outcome at each location. The temptation to think that this inevitably results in patterns which always appear simple should be resisted. There is now a substantial literature using cellular automata models to show how simple rules of spatial behaviour, combined with a discrete dynamical systems approach, can generate extremely complex patterns (see Von Neumann, 1966; Toffoli and Margolus, 1987). The approach was introduced into geography by Couclelis (1985) and has been developed in a GIS content by Camara and Castro (1996) and Sanders (1996). By their very nature, these models are readily implemented in the framework of a raster GIS, and there is a strong link between them and the world of fractals, nonlinear dynamics and 'chaos' theory.

It is, however, more usual to think of spatial processes as being stochastic, in which the outcome at each location or area represents a probabilistic (random) selection from some underlying generating distribution. The patterning in the spatial phenomena that results usually arises as a result of two types of variation, called first and second-order effects. First-order effects relate to variations in the mean value, or intensity, of the process over space. The process is spatially stationary if this intensity is constant over the study area. Second-order effects involve relationships between objects in the study area.

## III Detecting spatial pattern

Clearly, this statistical notion of pattern depends upon our having some standard against which to judge the spatial arrangement of data and the usually adopted one is that of complete spatial randomness (CSR – Diggle, 1983). In this approach pattern is equated with spatial homogeneity and inhomogeneity resulting from departures from CSR. In the first case, the underlying distribution function of the postulated process remains unchanged over space. In the second it changes in some way from place to place. Although most work has been done using point processes, the same notion of CSR can be applied to data describing line, area and surface objects (Unwin, 1981). A number of methods have been developed to enable patterns in spatial data to be described.

### 1 Global statistics

Global statistics attempt to characterize the patterning across an entire region. In ecology, geography and geology, a popular example is the Clark and Evans nearest neighbour index, calculated as the ratio of the observed mean distance from each point object to its nearest neighbour to the value expected from a CSR process with the same intensity. In effect this collapses the pattern on to a single dimension given by the X-value. A similar projection on to two axes is given by more recently developed methods, such as those which use the G, $F$ and $K$ functions to test for CSR in a point pattern (see Bailey and Gatrell, 1995). The G function assembles the complete cumulative probability

distribution of point event to point event nearest neighbour distances as a function of distance, *d:*

$$G(d) \ = \ \# \, (w_i < d) \, / \, n$$

In which # means 'the number of' and w are the nearest neighbour distances. Similarly, the *F* function does the same thing for distances between a randomly selected point in the region and the point events:

$$F(x) = \# \, (x_i < x) \, / \ m$$

in which *m* equals the number of point samples. These functions can be estimated from an observed point pattern and the general strategy is to compare the shape of the actual curve with that given by a CSR process with the same average intensity. Both are useful for examining small scales of pattern and both are a definite advance on the uncritical use of the simple nearest neighbour value. To examine pattern over a wide range of scales use is made of the *K* function, defined as:

$$\lambda \, . \, \mathrm{K} \, (d) = \mathrm{E} \, (\# \ \text{events within distance d of an arbitrary event})$$

in which λ is the average intensity. For a recent, accessible overview of these statistics and their usefulness in the analysis of point patterns in a GIS environment, see Gatrell *et al.* (1996). For area enumerated data a number of approaches have been adopted, ranging from simple spatial moving averages and smoothing by median polish to the computation of correlograms to show the spatial autocorrelation at differing adjacency and distance lags (Cliff and Ord, 1973). For visualizing such data the problem of low observed numbers, particularly in spatial epidemiology, has been addressed by the careful use of Poisson probability mapping (Langford, 1994), and the equally difficult problem of correction for variations in the 'weight of evidence' that arises naturally from the use of differently sized spatial units has been solved by the application of Bayes' theorem to derive appropriately weighted estimates (Marshall, 1991). Finally, for spatially continuous data, methods from geostatistics, notably use of the correlogram and (semi)variogram have become virtually standard (see Isaaks and Srivastava, 1989; Cressie, 1991). A particularly useful display is the semi-variogram cloud devised by Haslett and others (Haslett *et al.,* 1991).

  Global statistics have severe limitations for work in spatial data analysis. First, the assumption is usually that the pattern – and hence the process – is stable, or *stationary,* over space. As GIS systems have enabled researchers to use either larger study regions or, equivalently, data sets at much finer spatial resolution, so this assumption seems more and more unrealistic. Basic geographical theory shows that such spatial homogeneity over large areas of the earth's surface or at fine resolution is extremely unlikely (Fotheringham et al., 1996). What may well happen is that large areas of uninteresting spatial variation swamp those of real interest. Secondly, almost all these global measures are subject to potentially severe edge effects in their calculation and, thirdly, the expectations are frequently subject to the so-called modifiable area unit (MAU) problem (see, for example, Fotheringham and Wong, 1991; Fotheringham *et al.,* 1995). Taking the Clark and Evans X-index as example, its value depends to a large extent on the area chosen over which to study and hence the assumed mean *intensify* of the process.

Many years ago, in an attempt to show that within drumlin fields the distribution of these landforms is spatially random, I misguidedly computed the Clark and Evans nearest neighbour statistic (my collaborator is blameless, see Smalley and Unwin, 1968). It should be abundantly clear that, simply by redefining the study area, I could have produced almost any R-index. As it was by total (mis)chance I seemed to hit on the scale of analysis that produced values close to the magical random expectation.

Similar, much more subtle dependencies occur in virtually all the work that we do. In the example given, the use of a measure of pattern based on an inter-event distance is reasonable (although there will be edge effect problems) and the technical fault would nowadays easily be solved by randomization, but the same MAU problem is present in most of the tests for spatial autocorrelation where it appears first in the zonal aggregate values used and, secondly, in their conversion into standard scores based on some arbitrary global mean, but this problem is seldom mentioned by practitioners. Finally, and as a consequence of the considerations outlined above, where formal statistical tests are employed, the assumptions employed are almost invariably broken.

## 2 Visualization

In response to these problems, much exploratory spatial data analysis has turned to visualization as a means of pattern detection, the notion being that the eye/ brain system, when given sufficient help, is capable of a high degree of sophisticated pattern recognition. This is the philosophy of SPIDER/REGARD (Haslett et *al.,* 1990), cdv (Dykes, 1995) and a system based on XLispStat (Brunsdon and Charlton, 1995). The philosophy is that we use data display as a means of analysis in its own right and the problem becomes one of designing appropriate and useful types of display.

As outlined in my 1994 report (Unwin, D.J., 1994), pure visualization has its adherents and critics. First, it is well known in the literature on cartographic communication that apparently quite minor changes to a map can greatly change how it is viewed, a good example being the choice of class intervals in choropleth mapping. More subtle, but none the less important examples occur in any contour mapping and in almost all the use of colour coding. Very few visualization practitioners, at least in the Anglo-Saxon world, would agree with Bertin's notion of the monosemic (single sign) map, preferring instead to think of maps as polysemic (capable of many interpretations) products of frequently fallible cartographers. Secondly, it is also well known that the eye /brain frequently synthesizes a pattern where, strictly, the data are random. Similar effects have been seen where test groups produce different maps of the same numbers according to the information they are given about the phenomenon being mapped.

## 3 Local statistics

A third strategy is to harness the power of simple statistical summary of the type employed when global statistics are used to define pattern with the less formal, but equally less demanding, process of visualization by mapping what have been termed *local* statistics. Typically, in using local statistics we attempt to learn more about each individual datum relating to a point, line or area object in the data set by comparing it in some way to the values for its neighbouring objects. Several local statistics have been suggested and their use illustrated.

Getis and Ord (1992) define a G-function (which is not the same as the function of

the same name used in point pattern analysis, see above) which gives an index of spatial clustering of a set of observations over a defined neighbourhood:

$$G_i(d) = \Sigma \, w_{ij}(d).x_j \, / \, \Sigma \, x_j$$

In this, x is the regional variable and W(d) is a symmetric 0 / 1 matrix of weights with 1s for all the areas defined to be within distance, d, of the given area, $i$. All other elements are zero, including the link of $i$ to itself which means that the value for the area is not considered. Readers who have struggled to compute by hand one or other of the global statistics mentioned above will appreciate that the computation of G is hardly possible without the computer and access to a GIS type of data structure from which to determine the $W(d)$.

In effect, the vector of values for each region, *G(d),* shows how locally anomalous the region is with increasing distance for the given variable, x. Each area object in the data has its associated G-function which can be mapped for given *d,* or plotted as a function of distance. A restriction on this statistic is that as defined it is only useful if the variable x has a natural origin. It is thus inappropriate for the study of change variables or variables that have negative values.

Ord and Getis (1995) and, more recently, Bao and Henry (1996) develop a distribution theory for this statistic under the hypothesis of a random allocation of values which enables a standardization to give Z(G). In practical applications, where the number of zones in a neighbourhood is very low, the exclusion of the point- itself can give awkward problems, and a variant, G", is sometimes calculated in the same way but including the zone's own value. As with the standard form, so it is possible to calculate a standard score as $Z(G^*)$. These statistics are used to detect possible nonstationarity in data, where clusterings of similar values are found in specific subregions of the area studied.

As an alternative to the G statistics, Anselin (1993) has shown that the spatial autocorrelation coefficients Moran's I and Geary's contiguity ratio C can be decomposed into local values. The local form of Moran's I is a product of the zone value and the average in the surrounding zones:

$$I_i(d) = z_i. \, \Sigma \, w_{ij}(d).z_j$$

In making these calculations, the observations, $z$, are in standardized form, the W(d) matrix is row-standardized, and the summation is for all $j$ not equal to $i$. Bao and Henry (1996) show that under the conditional assumption of nonrandom observation at $i$, this local Moran is a linear transformation of the G statistic. Finally, and in fact rather different from G and $I$, Anselin (1995) outlines a local variant of Geary's contiguity ratio as:

$$C;(d) = \Sigma \, w_{ij}(d).(z_i - z_j)^2$$

In its use of the differences between the location value and its neighbours, this local form of C is in many ways similar to the gradient operators used in image processing.

The idea of a local statistic is not new and similar calculations have been routine in image processing for decades. In the restricted world of the raster GIS, Tomlin's analytical operations of the focal type are a second example (Tomlin, 1990). Recent work by Moore (1996) has shown how operations from image processing can be generalized

within a GIS to apply to irregular grids of polygonal areas or *resels* (resolution elements). Similarly, once we have the computer power, almost any of the classical statistics can be calculated as a local value (for example, the mean, standard deviation and correlation). This idea has recently been exploited by Fotheringham *et* al. (1996) who compute maps of how the estimated parameters of a regression model vary spatially over a fine raster of grid cells. Maps of these estimates provide additional information about the spatial stationarity of the model of the process which is complimentary to more conventional maps of residuals over a global regression.

## IV Defining localities

No matter what the statistic, a key question that must be addressed in these operations is the definition of what is meant by local. Depending on the definition adopted every location in the database will generate different statistics, and, as can be seen from the formal definitions of the local forms of $I$, G and C, use is made of a weights matrix W(d) representing all possible 0.5 $n*(n-1)$ interobject pairs. If each element, $w_{ij}$, is given the value 0 or 1 this forms an adjacency matrix which contains all the information needed to define the concept of local by adjacency. It is easily shown that successive powers of this matrix (with zeros down the diagonal, see Garner and Street, 1978) give the objects that are adjacent but two steps away, and so on, permitting an easy generalization of the notion of local. An early article describing the use of these extended neighbourhoods is Lebart (1969). For an example of the information content of powers of this adjacency matrix, see Unwin (1981: 87-93). In addition to the simple fact of adjacency (O/l ), the same matrix can be used to record the strength of the adjacency by modification of the $w_{ij}$ to record, for example, the length of common boundary.

Alternatively, metric distances can be used to define local in any one of several ways to provide a *kernel* around the location of interest (see Bailey and Gatrell, 1995: 261-62; Moore, 1996). The simplest option is to set the relevant $w_{ij}$ equal to 0 or 1 according to whether or not the zone centroids are within some distance, *d,* of each other. Alternatively, use can be made of inverse distance weighting according to some function as in statistical density estimation (Silverman, 1986) or spatial interpolation using algorithms based on the old SYMAP scheme in what Fotheringham et al. (1996) call spatial regression. Although one could use all objects in the database in such a weighting, in practice attention is usually confined to a restricted kernel width but it may be that this restriction is unnecessary in these days of very high-performance machines. A third alternative is to use an adaptive kernel which is responsive to the local data density or to optimize the distance used in some way. An obvious approach is to use the range, as deduced from estimation of the semi-variogram, as the kernel width, but there is scope here for experimentation with other criteria based, for example, on kernel widths which maximize or minimize the local variation (i.e., the local geography). In the long run, it may well be that the most valuable information is contained in the behaviour of these local statistics as the kernel width is expanded. Preliminary work by Wood (1996) using local regression results to provide multiscale characterizations of landform from digital elevation matrices suggests that this scale dependence contains useful geographical  information.

## V **Conclusion: putting spatial statistics into GIS**

It should be clear that, influenced by GIS, the availability of very large, high spatial resolution data, and access to extremely powerful computer power, spatial data analysis has already changed greatly and will continue to change as methods which recognize the existence of today's data and computer-rich environment are developed.

A concept that has been developed by Openshaw and his colleagues (see Openshaw and Clarke, 1996) is that of *GISable* statistics (see Table 1). By this they mean analytical and other approaches that are suited to a world in which computer power, very large data sets and the availability of GIS should be taken for granted. The concept is a useful one, since, as they point out, it helps define a research agenda for developing new methods and draws attention to the fundamentally unsatisfactory, even unsound, nature of the traditional methods of statistical analysis when applied to spatial data. For example, almost all the spatial statistical methods I included in Introductory *spatial analysis* (Unwin, 1981) should have very little place in our current research environment. Almost without exception they can be replaced by more recent techniques or, as is Openshaw's predilection, by a variety of compute intensive procedures.

Already, there is a long list of possible GISable statistical functions that might be added to improve the functionality for spatial statistical analysis of existing GIS. We all have our special favourites but, on the basis of the methods I generally find myself persuading MSc and PhD students to adopt, my own list would include the following:

- All the methods for the analysis of point events developed by statisticians in the 40 years since that dreadful 'nearest neighbour' statistic was first proposed.
- The spatial form of density estimation in the manner of Silverman (1986) and as demonstrated by Gatrell (1994).
- Generalized linear modelling tools, in the manner of GLIM (Aitkin et al., 19891, to be used for calibrating various forms of favourability functions in map overlay (Bonham-Carter, 1991).
- A series of local statistical indicators of spatial association and inhomogeneity of the type outlined above.
- The ability easily to change and visualize W(d).

Table **1**    Ten rules for developing 'GISable' statistical analysis

| | |
|---|---|
| *Rule* 1 | A GISable spatial analysis method should be able to handle large and very large N values |
| *Rule 2* | Useful GISable analysis and modelling tools are study region independent |
| *Rule 3* | GIS relevant methods need to be sensitive to the special nature of spatial information |
| *Rule 4* | The results should be mappable |
| *Rule 5* | GISable spatial analysis is generic |
| *Rule 6* | GISable spatial analysis methods should be useful and valuable |
| *Rule 7* | Interfacing issues are initially irrelevant and subsequently a problem for others to solve |
| *Rule 8* | Ease of use and understandability are very important |
| *Rule 9* | CISable analysis should be safe technology |
| *Rule 10* | GIS methods should be useful in an applied sense |

*Source:* Adapted from Openshaw and Clarke (1996).

- Good exploratory visualization tools of the type proposed by Densham (1994) and offered by REGARD (Unwin, A.R., 1994) and cdv (Dykes, 1996).

There may well be others, but the general nature of my list should be clear. It attempts to update what most geographers think of as spatial statistical analysis and thus correct the evident inability of many to take advantage of almost all the developments in statistics, spatial statistical analysis and computing since the 1960s.

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